# Symmetry Friend or Foe: Confluences of Conical Intersection Seams in Tetra-Atomic Molecules ${ }^{\dagger}$ 

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#### Abstract

It is usually assumed that seams of conical intersection are isolated features, that is, a displacement perpendicular to the seam leads to a removal of the degeneracy. However, this need not be the case. It has been shown that for triatomic molecules, there may exist a confluence of an accidental symmetry-allowed $C_{2 v}$ seam of conical intersection with an accidental $C_{s}$ seam of conical intersection of two states of the same symmetry. Here, we demonstrate the existence of a related confluence of seams of conical intersection in the tetra-atomic molecule isocyanic acid. Such confluences have not been previously reported. A perturbative analysis of the vicinity a seam of conical intersection is used to explain origin of this novel locus of conical intersection points.


## I. Introduction

Points of conical intersection are continuously connected, forming seams. The seam is a surface of dimension $N^{\text {int }}-2$, where $N^{\text {int }}$ is the number of internal coordinates. ${ }^{1}$ Perpendicular to the seam is a plane, the $\mathrm{g}-\mathrm{h}^{2}$ or branching ${ }^{3}$ plane, in which the energy has the form of a double cone, that is, the degeneracy is lifted in a linear manner. Because the dimension of the seam plus that of the $\mathrm{g}-\mathrm{h}$ plane is $N^{\text {int }}$ seams are expected to be isolated features, where isolated implies that there exists a volume that contains this and only this seam. Recently, it was shown that this is not always the case in triatomic molecules. ${ }^{4-7}$ In some triatomic molecules, all of which had $C_{2 v}$ or higher symmetry, an intersection or confluence of a symmetry-allowed seam of conical intersection, and a seam of conical intersection of two states of the same-symmetry (a same-symmetry seam) was found. We showed how these confluences could be anticipated using the cross product of the two vectors that define the $\mathrm{g}-\mathrm{h}$ plane. ${ }^{8,9}$ Subsequently, intersections of three symmetry equivalent, symmetry-allowed seams, with one symmetryrequired seam were found in $X_{3}$ molecules. ${ }^{10,11}$

In a triatomic molecule, the seams are lines so that identifying the two seams that form the intersection is straightforward. The situation is more complex in molecules with more than three atoms. Here, the seam is a generalized line of dimension $N^{\text {int }}-$ $2 \geq 4$. Thus, it is not possible to say by inspection whether a pair of adjacent points of conical intersection belong to the same or distinct seams of conical intersection. Here, we will refer these distinct seams as distinct branches of an overall seam to emphasize the fact that the same two electronic states are involved throughout.

In this work, a previously reported ${ }^{12}$ perturbative description of the vicinity of a point of conical intersection is used analyze the symmetry-allowed seam of conical intersection between an $A^{\prime}$ and an $A^{\prime \prime}$ state in a tetra-atomic molecule. It is shown for this symmetry-allowed seam of conical intersection a confluence with a same-symmetry seam is possible. Criteria for identifying points on the distinct branches of the seam of conical intersection are established. Several interesting aspects of the analysis reflect

[^0]the fact that a coplanar arrangement of four atoms has only one totally nonsymmetric mode in $C_{s}$ symmetry.
In Section II, perturbation theory is used to develop criteria for identifying points on distinct branches of a seam of conical intersection near their confluence. In Section III, these criteria are used to demonstrate the existence of intersecting branches for the $S_{1}-S_{0}$ seam of conical intersection in HNCO. The identification of this confluence is particularly timely as radiationless decay of the $\mathrm{S}_{1}\left[1^{1} \mathrm{~A}^{\prime \prime}\left(2^{1} \mathrm{~A}\right)\right]$ state of HNCO , which is attributable to the $S_{1}-S_{0}$, seam of conical intersection, has recently been a topic of considerable interest. ${ }^{13-17}$ Section IV provides a summary and our conclusions.

## II. Theory

The adiabatic states $\Psi^{a}(\mathbf{r} ; \mathbf{R})$ eigenfunctions of the electronic Schrödinger equation

$$
\begin{equation*}
\left[H(\mathbf{r} ; \mathbf{R})-E_{\mathrm{K}}(\tau)\right] \Psi_{K}^{a}(\mathbf{r} ; \mathbf{R})=0 \tag{1}
\end{equation*}
$$

may be expressed in a basis of symmetry-adapted configuration state functions (CSFs, $\boldsymbol{\psi})^{18}$ as

$$
\begin{equation*}
\Psi_{K}^{a}(\mathbf{r} ; \mathbf{R})=\sum_{\alpha=1}^{N_{\alpha}^{\mathrm{CSF}}} c_{\alpha}^{K}(\tau) \psi_{\alpha}(\mathbf{r} ; \mathbf{R}) \tag{2a}
\end{equation*}
$$

so that the $\mathbf{c}^{\mathrm{K}}$ satisfy

$$
\begin{equation*}
\left[\boldsymbol{H}(\tau)-E_{\mathrm{K}}(\tau)\right] \mathbf{c}^{K}(\tau)=0 \tag{2b}
\end{equation*}
$$

Here, $\mathbf{r}$ denotes the $3 \mathrm{~N}^{\text {el }}$ electronic coordinates, $\mathbf{R}$ denotes the $3 N^{\text {nuc }}$ space-fixed nuclear coordinates, and $\tau$ denotes $N^{\text {int }}$ intersection adapted internal nuclear coordinates. ${ }^{3}$ Intersection adapted coordinates are a set of generalized cylindrical polar coordinates with origin $\tau_{x}$, a point of conical intersection of states $I$ and $J$, and $(x, y)$-plane given by the branching ${ }^{3}$ or $\mathrm{g}-\mathrm{h}^{2}$ plane which is spanned by the tuning mode ${ }^{19} \mathbf{x}=\mathbf{g}^{\mathrm{IJ} / g}$, and the coupling mode $\mathbf{y}=\mathbf{h}^{\mathrm{IJ}} / \mathrm{h}$, where $g=\left\|\boldsymbol{g}^{\mathrm{IJ}}\right\|, h=\left\|\boldsymbol{h}^{\mathrm{IJ}}\right\|$, and

$$
\begin{equation*}
2 g_{\tau}^{I J}(\tau)=-\mathbf{c}^{I}\left(\tau_{x}\right)^{\dagger} \frac{\partial \boldsymbol{H}(\tau)}{\partial \tau} \mathbf{c}^{I}\left(\tau_{x}\right)+\mathbf{c}^{J}\left(\tau_{x}\right) \frac{\partial \boldsymbol{H}(\tau)}{\partial \tau} \mathbf{c}^{J}\left(\tau_{x}\right) \tag{3a}
\end{equation*}
$$

$$
\begin{equation*}
h_{\tau}^{I J}(\tau)=\mathbf{c}^{I}\left(\tau_{x}\right)^{\dagger} \frac{\partial \boldsymbol{H}(\tau)}{\partial \tau} \mathbf{c}^{J}\left(\tau_{x}\right) \tag{3b}
\end{equation*}
$$

The remaining $N^{\text {int }}-2$ mutually orthogonal internal coordinates, $\mathbf{z}^{i}, i=1-\left(\mathrm{N}^{\text {int }}-2\right)$, describe the seam. In these coordinates, $\boldsymbol{H}^{\mathrm{d},(2)}(\tau)$, the electronic Hamiltonian for states $I$ and $J$ through second order in the displacement from $\tau_{\mathrm{x}}$ is given by

$$
\begin{equation*}
\boldsymbol{H}^{\mathrm{d},(2)}(\tau)=\Sigma^{(2)}(\tau) \boldsymbol{I}+[-g x+A(\tau)] \sigma_{z}+[h y+B(\tau)] \sigma_{x} \tag{4}
\end{equation*}
$$

where $\sigma_{x}, \sigma_{y}, \sigma_{z}$ are the Pauli spin-matrices

$$
\begin{gather*}
\Sigma^{(2)}(\tau)=s_{x} x+s_{y} y+\sum_{i} s_{z} z^{i}+N(\tau)  \tag{5a}\\
s_{\tau}^{I J}(\tau)=\mathbf{c}^{I}\left(\tau_{x}\right)^{\dagger} \frac{\partial \boldsymbol{H}(\tau)}{\partial \tau} \mathbf{c}^{I}\left(\tau_{x}\right)+\mathbf{c}^{J}\left(\tau_{x}\right) \frac{\partial \boldsymbol{H}(\tau)}{\partial \tau} \mathbf{c}^{J}\left(\tau_{x}\right) \tag{5b}
\end{gather*}
$$

and

$$
\begin{align*}
A(\tau) & =\left(a_{1}^{(\rho)} x^{2}+a_{2}^{(\rho)} y^{2}+a_{3}^{(\rho)} x y\right)+\sum_{i}\left(a_{1}^{\left(z_{i}\right)} x z_{i}+a_{2}^{\left(z_{i}\right)} y z_{i}\right)  \tag{6a}\\
& \equiv \rho^{2} A^{(\rho)}(\theta)+\rho \sum_{i} z_{i} A^{\left(z_{i}\right)}(\theta) \\
B(\tau) & =\left(b_{1}^{(\rho)} x^{2}+b_{2}^{(\rho)} y^{2}+b_{3}^{(\rho)} x y\right)+\sum_{i}\left(b_{1}^{\left(z_{i}\right)} x z_{i}+b_{2}^{\left(z_{i}\right)} y z_{i}\right)  \tag{6b}\\
& \equiv \rho^{2} \mathrm{~B}^{(\rho)}(\theta)+\rho \sum_{i} z_{i} B^{\left(z_{i}\right)}(\theta) \\
N(\tau) & =\left(n_{1}^{(\rho)} x^{2}+n_{2}^{(\rho)} y^{2}+n_{3}^{(\rho)} x y\right)+\sum_{i}\left(n_{1}^{\left(z_{i}\right)} x z_{i}+n_{2}^{\left(z_{i}\right)} y z_{i}\right)+ \\
& \equiv \sum_{i, j} n_{3}^{\left(z_{i} z_{j}\right)} z_{i} z_{j} \\
& =\rho^{2} \mathbf{N}^{(\rho)}(\theta)+\rho \sum_{i} z_{i} N^{\left(z_{i}\right)}(\theta)+\mathrm{N}^{\left(z_{z}, z^{2}\right.}
\end{align*}
$$

The second-order contributions $A(\tau), B(\tau)$, and $N(\tau)$ are the most general form consistent with a piecewise linear representation of the seam. With the exception of $n_{i}{ }^{(w)}$, these coefficients can be determined from the circulation, the line integral ${ }^{20}$ along a closed loop, of the derivative coupling. ${ }^{12}$

For this ${ }^{1} \mathrm{~A}^{\prime}-{ }^{1} \mathrm{~A}^{\prime \prime}$ seam of conical intersection, we can, without loss of generality, require $\mathbf{x} \sim a^{\prime}$ and $\mathbf{y} \sim a^{\prime \prime}$, where $\sim$ means "transforms as". This symmetry requires some of the above constants to vanish so that

$$
\begin{gather*}
\mathrm{A}(\tau)=\left(a_{1}^{(\rho)} x^{2}+a_{2}^{(\rho)} y^{2}\right)+\sum_{i} a_{1}^{\left(z_{i}\right)} x z_{i}  \tag{7a}\\
B(\tau)=b_{3}^{(\rho)} x y+\sum_{i} b_{2}^{\left(z_{i}\right)} y z_{i} \tag{7b}
\end{gather*}
$$

As a consequence of these symmetry-imposed constraints and the fact that there is only one $a^{\prime \prime}$ mode, $H_{I J}^{(2), \mathrm{d}}=h \mathrm{y}+B(\tau)$ is factorable, $H_{I J}^{(2), \mathrm{d}}=H^{i} H^{i i}$ where

$$
\begin{gather*}
H^{\mathrm{i}}(x, y, z)=y  \tag{8ai}\\
H^{\mathrm{ii}}(x, y, z)=h+b_{3}^{(\rho)} x+\sum_{i} b_{2}^{\left(z_{i}\right)} z_{i} \tag{8aii}
\end{gather*}
$$

This factorization is key. As a result of the factorization, the
conditions for a seam of intersection $H_{I J}^{(2), \mathrm{d}}=G=0$, where

$$
\begin{equation*}
G(x, y, z)=\left(a_{1}^{(\rho)} x^{2}+a_{2}^{(\rho)} y^{2}\right)+\sum_{i} a_{1}^{\left(z_{i}\right)} x z_{i}-g x \tag{8b}
\end{equation*}
$$

yields two pair of equations, that is, the seam consists of two branches, $S(i)$ and $S(i i)$ where points on $S(i)$ satisfy $H^{i}=G=$ 0 and those on $\mathrm{S}(i i)$ satisfy $H^{i i}=G=0$.
$S(i)$ is confined to the subspace $y=0$, the $C_{s}$ symmetry subspace. Therefore, any nonplanar $\tau_{x}$ necessarily belongs to $S(i i)$. From eq 4, the coupling mode for $S(i)$ is perpendicular to the molecular $(\mathrm{y}=0)$ plane, so that it has $a^{\prime \prime}$ symmetry. Thus, $S(i)$ represents a symmetry-allowed seam of two states of different symmetry. $S(i i)$, on the other hand, is not confined to the space $y=0$. It can exist for non planar geometries, where it necessarily constitutes a same symmetry intersection. The observations in this paragraph provide a criterion (criterion I) for determining points on $S(i i)$.

The seam equations, $H^{i i}=G=0$ and $H^{i}=G=0$ can be solved to find the locus of points of confluence of conical intersections. Because $G=0$ is common to both seams, the solution space for the confluence has dimension $N^{\text {int }}-3$. The confluence, if it exists, occurs for $y=0$, that is, for planar geometries. The point $(x, 0, z)$ lies at a confluence provided

$$
\begin{gather*}
-\left(h+\sum_{i} b_{2}^{\left(z_{i}\right)} z_{i}\right) / b_{3}^{(\rho)}=x  \tag{9a}\\
x\left[\left(a_{1}^{(\rho)} x\right)+\left(-g+\sum_{i} a_{1}^{\left(z_{i}\right)} z_{i}\right)\right]=0 \tag{9b}
\end{gather*}
$$

In this work, we do not solve eq 9 a and 9 b to locate a confluence, using instead a procedure based on knowledge of $\mathbf{g}^{\mathrm{IJ}} \times \mathbf{h}^{I J}$. See Section III and Refs 8 and 21 .

Equation 8, parts a and b, can also be used to construct $\nabla G$ and $\nabla H_{I J}^{(2), \mathrm{d}}$ for seam $S(i)$ and seam $S(i i)$

$$
\begin{gather*}
\nabla G=\left(-g+a_{1}^{(\rho)} x+\sum_{i} a_{1}^{\left(z_{i}\right)} z_{i}\right) \boldsymbol{i}, 2 a_{2}^{(\rho)} y \boldsymbol{j}, a_{1}^{\left(z_{i}\right)} x \boldsymbol{k}_{i}  \tag{10a}\\
\nabla H^{i}=\boldsymbol{j} \quad \nabla H^{i i}=b_{3}^{(\rho)} \boldsymbol{i}, 0 \boldsymbol{j}, b_{2}^{\left(z_{i}\right)} \boldsymbol{k}_{i} \tag{10b}
\end{gather*}
$$

From eq 10 b, the vectors $\nabla \mathrm{H}^{l}, \nabla \mathrm{H}^{u}$ are seen to be orthogonal. ${ }^{22}$ This then is a criterion (criterion II) for identifying points belonging to distinct branches of a seam of conical intersection in the neighborhood of their confluence. Noting that for wave functions, given by eq $2, \nabla G \rightarrow \mathbf{g}^{I J}$ and $\nabla H_{I J}^{(2), \mathrm{d}} \rightarrow \mathbf{h}^{I J}$, and that these directions are orthogonal to the $\mathbf{z}^{i}$, the seam coordinates, this criterion can be rephrased as

Two adjacent points $\tau_{x}(k), k=i, j$, of conical intersection are associated with distinct branches of a seam of conical intersection provided either $\mathbf{g}^{I J}\left(\tau_{x}(i)\right)$ or $\mathbf{h}^{I J}\left(\tau_{x}(i)\right)$ can be expanded in terms of $\mathbf{z}^{i}\left(\tau_{x}(j)\right)$, the seam directions for $\tau_{x}(j)$

This criterion will be applicable to analogous confluences in molecules with more than four atoms, for which, however, factorization of $H_{I J}^{(2), \mathrm{d}}$ cannot be guaranteed. Confluences of seams of conical intersection have not been reported for tetraatomic molecules. Below, the existence of a confluence of this type is demonstrated in HNCO.

## III. Intersection Seams of Conical Intersection in HNCO

The excited $\mathrm{S}_{1}$ state of HNCO has both cis and trans minima, although only the trans minimum has been observed. Following photoexcitation from $\mathrm{S}_{0}, \mathrm{HNCO}$ in $\mathrm{S}_{1}$ can decay to groundstate $H+N C O$. Direct decay on $S_{1}$ is precluded by a large


Figure 1. (a) In the molecular ( $X, Y$ ) plane: $\tau_{\text {eq }}($ cis $)$ filled circles; $\tau_{x}($ cis, $2.65, \mathrm{p})$ open circles and scaled $\mathbf{g}^{\mathrm{ID}}$ filled arrows. (b) In the ( $X, Z$ ) plane scaled $\mathbf{h}^{\mathrm{JJ}}$ filled arrows.
barrier, necessitating a radiationless transition to $\mathrm{S}_{0}$ prior to dissociation. ${ }^{15,16}$ To describe this nonadiabatic transition, we have previously determined the portions of the $S_{1}-S_{0}\left(1^{1} \mathrm{~A}-\right.$ $2^{1} \mathrm{~A}$ ) seam of conical intersection for both cis and trans arrangements of HNCO as a function of $R(\mathrm{C}-\mathrm{N})$ using a large (approximately 3.5 million) CSF expansion. ${ }^{23}$ Both planar and nonplanar structures were found. These points of conical intersection were denoted $\tau_{x}(\mathrm{w}, R(\mathrm{C}-\mathrm{N}), \mathrm{z})$ where $\mathrm{w}=\mathrm{cis}$ or trans, and $z=\operatorname{planar}(\mathrm{p})$ or nonplanar (np). For the planar, $C_{s}$, geometries encountered in that study, the ground and excited states are of ${ }^{1} \mathrm{~A}^{\prime}$ and ${ }^{1} \mathrm{~A}^{\prime \prime}$ symmetry, respectively. In this case $\mathbf{g}^{\mathrm{IJ}}\left[\mathbf{h}^{\mathrm{IJ}}\right]$ has $a^{\prime}\left[a^{\prime \prime}\right]$ symmetry.

Figure 1a and 1 b depicts $\tau_{x}(\mathrm{cis}, 2.65, \mathrm{p}$ ) the planar (all atoms in the ( $X, Y$ ) plane) point of conical intersection with $R(\mathrm{C}-\mathrm{N})$ $=2.65 a_{0}$ together with the corresponding $\mathbf{g}^{\mathrm{IJ}}$ and $\mathbf{h}^{\mathrm{IJ}}$. Note that $\mathbf{g}^{I J}$ is in the molecular (X,Y) plane, whereas $\mathbf{h}^{I J}$ is perpendicular

TABLE 1: Points on Two Intersecting Branches of $\mathbf{1}^{\mathbf{1}} \mathrm{A}-$ $2^{1}$ A Seam of Conical Intersection ${ }^{a}$

| $R(\mathrm{C}-\mathrm{N})$ | $R(\mathrm{~N}-\mathrm{H})$ | $R(\mathrm{CO})$ | $\angle \mathrm{HNC}$ | $\angle \mathrm{NCO}$ | $\angle \mathrm{HNCO}$ | $E$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S(i i):$ same symmetry branch |  |  |  |  |  |  |
| 2.66 | 1.944 | 2.288 | 103.4 | 105.9 | -2.4 | 5952.1 |
| 2.67 | 1.955 | 2.271 | 103.7 | 106.1 | -3.9 | 5946.5 |
| $S(i):$ symmetry allowed branch |  |  |  |  |  |  |
| 2.65 | 1.940 | 2.375 | 111.2 | 109.4 | 0 | 4529.3 |
| ${ }^{a}$ Energies in cm |  |  |  |  |  |  |
| -168.098 | 267 au. |  |  |  |  |  |



Figure 2. $\mathbf{g}^{\mathrm{IJ}} \times \mathbf{h}^{\mathrm{IJ}} \equiv g h \sin \alpha$, where $\alpha$ is the included angle, and $h$ is plotted vs $R(\mathrm{C}-\mathrm{N})$. The vanishing of $h$ indicates an intersection of branches of a seam of conical intersection.
to that plane, that is, they have $a^{\prime}$ and $a^{\prime \prime}$ symmetry, respectively, as expected. To demonstrate that the points of conical intersection considered here do not occur in an irrelevant region of nuclear coordinate space, Figure 1a compares $\tau_{x}$ (cis, 2.65, p) with the cis- equilibrium structure on $\mathrm{S}_{1}, \tau_{\mathrm{eq}}(\mathrm{cis})$ and Table 1 reports the relevant energies.

Figure 2 plots $\mathbf{g}^{\mathrm{IJ}} \times \mathbf{h}^{\mathrm{IJ}}$ and $h$ for $\tau_{x}(\mathrm{cis}, R(\mathrm{C}-\mathrm{N}), \mathrm{np})$ as a function of $R(\mathrm{C}-\mathrm{N})$, for $R(\mathrm{C}-\mathrm{N})$ near $2.65 a_{0}$. From this plot, it is seen that $\mathbf{g}^{\mathrm{IJ}} \times \mathbf{h}^{\mathrm{IJ}}$ and $h$ vanish near $R(\mathrm{C}-\mathrm{N})=2.65 a_{0}$. This suggests a confluence of two branches of the $1^{1} \mathrm{~A}-2^{1} \mathrm{~A}$ seam of conical intersection in this region. According to the results of Section II, the confluence should consist of the symmetry-allowed $1^{1} \mathrm{~A}^{\prime}-1^{1} \mathrm{~A}^{\prime \prime}$ branch and a same-symmetry $1^{1} \mathrm{~A}-2^{1} \mathrm{~A}$ branch. For near $C_{s}$ geometries on the same symmetry branch, the coupling mode $\mathbf{h}^{I J}$ is predicted to have, approximately because the molecule is not planar, $a^{\prime}$ symmetry, a highly counterintuitive result, given the proximity of the ${ }^{1} \mathrm{~A}^{\prime}$ $-{ }^{1} \mathrm{~A}^{\prime \prime}$ conical intersection, where $\mathbf{h}^{I J} \sim a^{\prime \prime}$. This point is considered in Figure 3 which reports $\mathbf{g}^{I J}$ and $\mathbf{h}^{I J}$ for the slightly nonplanar point of conical intersection $\tau_{x}$ (cis,2.66,np). See Table 1. From Figure 3, both $\mathbf{g}^{I J}$ and $\mathbf{h}^{I J}$ are seen to be approximately contained in the "molecular plane", that is, again in an approximate sense, both $\mathbf{g}^{I J}$ and $\mathbf{h}^{I J}$ are of $a^{\prime}$ symmetry! Thus, $\mathbf{h}^{\mathrm{IJ}}\left(\tau_{x}(\right.$ cis $\left., 2.66, \mathrm{np})\right)$ is contained in the seam space of $\tau_{x}($ cis $, 2.65, \mathrm{p})$, and the converse is also true. This statement is quantified with the help of Table 2 which reports, in terms of atom-centered coordinates, the intersection adapted coordinates for $\tau_{x}(\mathrm{cis}, 2.66, \mathrm{np})$, in the order $\left(\mathbf{x}, \mathbf{y}, \mathbf{z}^{1}, \mathbf{z}^{2}, \mathbf{z}^{3}, \mathbf{z}^{4}\right)$. Note that $\mathbf{z}^{3}$ is the approximately


Figure 3. $\tau_{x}$ (cis, $2.66, n \mathrm{n}$ ) open circles, scaled $\mathbf{g}^{\mathrm{IJ}}$ filled arrows and scaled $\mathbf{h}^{\mathrm{IJ}}$ open arrows.

TABLE 2: Intersection Adapted Coordinates for $\boldsymbol{\tau}_{x}(\mathbf{c i s}, \mathbf{2 . 6 6}$, np ) in Terms of Atom Centered Coordinates (Atom A, Cartesian direction w)

| $\mathrm{Aw}^{a}$ | $\mathbf{g}^{I J}$ | $\mathbf{h}^{I J}$ | $\mathbf{z}^{1}$ | $\mathbf{z}^{2}$ | $\mathbf{z}^{3}$ | $\mathbf{z}^{4}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Hx | -0.011600 | -0.00980 | 0.83590 | 0.00000 | 0.00000 | 0.00000 |
| Hy | -0.002000 | 0.07430 | -0.12720 | 0.70820 | 0.00000 | 0.00000 |
| Hz | 0.011500 | 0.00100 | 0.01380 | -0.01380 | 0.44030 | 0.00000 |
| Nx | -0.519500 | 0.33260 | -0.23040 | 0.10170 | 0.02060 | 0.48400 |
| Ny | 0.103600 | 0.40350 | -0.090 | 20 | -0.65130 | -0.01710 |
| Nz | -0.032800 | -0.01590 | 0.00340 | 0.01050 | -.60280 | 0.00640 |
| Cx | 0.530700 | 0.35710 | -0.21600 | 0.10240 | -.03480 | -.46090 |
| Cy | -0.51350 | -0.14820 | 0.08140 | -0.09780 | 0.04060 | -.57700 |
| Cz | 0.048500 | 0.01740 | -0.01030 | 0.01290 | 0.54240 | 0.01290 |
| Ox | 0.000400 | -0.67990 | -0.38960 | -0.20410 | 0.01420 | -.02310 |
| Oy | 0.41190 | -0.32960 | 0.13590 | 0.04090 | -.02350 | 0.45100 |
| Oz | -0.027300 | -0.00250 | -0.00700 | -0.00960 | -.37990 | -.01930 |

$a^{\prime \prime}$ mode. In this basis, $\mathbf{h}^{1 \mathrm{~J}}\left(\tau_{x}(\operatorname{cis}, 2.65, \mathrm{p})\right)=(-0.2134,-0.0781$, $-0.0505,0.1164,-\mathbf{0 . 9 6 2 3}$, and -0.0582 ) and $\mathbf{g}^{I J}\left(\tau_{x}(\right.$ cis $\left.2.65, \mathrm{p})\right)$ $=(-\mathbf{0 . 9 0 4 2},-\mathbf{0 . 2 7 9 0}, 0.0642,0.0449,0.2107$, and 0.1983 $)$. The bold face components demonstrate that $\mathbf{h}^{\mathrm{LJ}}\left(\tau_{x}(\mathrm{cis}, 2.65, \mathrm{p})\right)$ is contained (largely) in the seam space of $\tau_{x}$ (cis, $\left.2.66, \mathrm{np}\right)$. This confirms that this same symmetry point of conical intersection $\tau_{x}($ cis, $2.66, \mathrm{np})$ and the symmetry-allowed point of conical intersection $\tau_{x}$ (cis, $2.65, \mathrm{p}$ ) reside on distinct branches of the $1^{1} \mathrm{~A}-2^{1} \mathrm{~A}$ seam of conical intersection.

Note that criterion II is only approximately satisfied. This is attributed to the fact the $\tau_{x}$ are not sufficiently close (see Table 1) for the perturbative result to hold preicsely. This in turn is a consequence of the small value of $h$ for points close to the confluence which sometimes makes locating such points difficult for our algorithm ${ }^{24}$ which requires $\mathbf{g}^{I J}$ and $\mathbf{h}^{I J}$ to be non vanishing. Because this difficulty occurs principally for the symmetry-allowed seam it can be overcome by computing the symmetry-allowed crossing using wave functions with $C_{s}$ symmetry imposed. ${ }^{25}$ This approach will be used in a future publication, in which a discussion of the trans portion of the seam and a more complete mapping of this confluence will be presented.

## IV. Summary and Conclusions

We report for the first time a confluence of two seams of conical intersection in a tetra-atomic molecule. The existence of the confluence is rationalized using perturbation theory. The confluence in question represents the intersection of an accidental symmetry-allowed conical intersection seam and an accidental same-symmetry conical intersection seam. The existence of intersecting seams of conical intersection significantly complicates the description of the overall intersection seam. It is, therefore, ironic to observe that the characterization of an accidental symmetry-allowed seam of conical intersection, here, the ${ }^{1} \mathrm{~A}^{\prime}-{ }^{1} \mathrm{~A}^{\prime \prime}$ seam, is greatly facilitated by fact that it is symmetry-allowed. In this case, the energies can be determined from separate blocks of the Hamiltonian matrix, making an avoided intersection due to configuration interaction is impossible. However, as described in detail in section II, it is precisely this symmetry that makes a confluence with a same-symmetry seam is possible.

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[^0]:    ${ }^{\dagger}$ Part of the special issue "William H. Miller Festschrift".

